SOLUTIONS

Solution 1 :

$$\mathcal{A} = \int_0^1 \frac{x^2}{\sqrt{1+2x^3}}\,dx = \int_0^1 x^2(1+2x^3)^{\frac{-1}{2}}\,dx = \frac{1}{6}\int_0^1 6x^2(1+2x^3)^{\frac{-1}{2}}\,dx$$

$$= \frac{1}{6}\cdot\left[\frac{(1+2x^3)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1 = \frac{\sqrt{3}-1}{3}$$

$$2°/\ \mathcal{A} = \int_0^1 t\sqrt{1-t^2}\,dt = \frac{1}{2}\int_0^1 2t(1-t^2)^{\frac{1}{2}}\,dt = \frac{1}{2}\left[\frac{(1-t^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{-1}{3}.$$

$$3°/\ \mathcal{A} = \int_{-1}^0 \frac{2t}{(t^2+1)^3}\,dt = \int_{-1}^0 2t(t^2+1)^{-3}\,dt = \left[\frac{(t^2+1)^{-2}}{-2}\right]_{-1}^0 = \frac{-3}{8}.$$

$$4°/\ \mathcal{A} = \int_0^1 \frac{1}{\sqrt{1+x}}\,dx = 2\left[\sqrt{1+x}\right]_0^1 = 2(\sqrt{2}-1).$$

$$5°/\ \mathcal{A} = \int_0^2 (1-|x-1|^3)\,dx = \int_0^1 (1-|x-1|^3)\,dx + \int_1^2 (1-|x-1|^3)\,dx$$

$$= \int_0^1 (1-(1-x)^3)\,dx + \int_1^2 (1-(x-1)^3)\,dx$$

$$= \left[x+\frac{(1-x)^4}{4}\right]_0^1 + \left[x-\frac{(x-1)^4}{4}\right]_1^2 = \frac{3}{4}+\frac{3}{4} = \frac{3}{2}$$

$$6°/\ \mathcal{A} = \int_0^{\frac{\pi}{3}} \cos x \sin^4 x\,dx = \left[\frac{\sin^5 x}{5}\right]_0^{\frac{\pi}{3}} = \frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)^5 = \frac{9\sqrt{3}}{160}.$$

$$7°/\ \mathcal{A} = \int_{\frac{\pi}{3}}^0 \cos^3 x\,dx = \int_{\frac{\pi}{3}}^0 \cos x\cos^2 x\,dx = \int_{\frac{\pi}{3}}^0 \cos x(1-\sin^2 x)\,dx$$

$$= \int_{\frac{\pi}{3}}^0 (\cos x - \cos x\sin^2 x)\,dx = \left[\sin x - \frac{\sin^3 x}{3}\right]_{\frac{\pi}{3}}^0 = \frac{-3\sqrt{3}}{8}$$